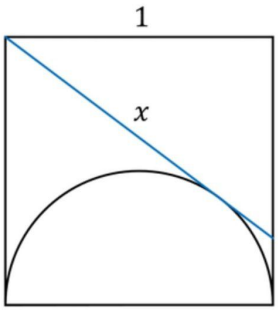
October Math Gems

Problem of the week 3

§1 problems

Problem 1.1. The length of the perimeter of a right triangle is 60 inches and the length of the altitude perpendicular to the hypotenuse is 12 inches. Find the sides of the triangle.

Problem 1.2. A square with the side length 1 has an inscribed semicircle along one of its sides as shown here, Find the value of x.



Problem 1.3. If $x + \frac{1}{16x} = 1$, then the value of

$$64x^3 + \frac{1}{64x^3}$$

is?

Problem 1.4. If $f(x) = ax^2 + bx + c$, and $f(2) = 0$, $f(3) = 11$, then find the value of

$$a - b - c =$$

Problem 1.5. If

$$a^4 + a^2b^2 + b^4 = 8$$

and

$$a^2 + ab + b^2 = 4$$

Then compute the value of

$$ab$$

?

Problem 1.6. If $x^2 + y^2 + z^2 = xy + yz + zx$, then compute the value of

$$\frac{4x + 2y - 3z}{2x}$$

Problem 1.7. Solve in R

$$\sqrt{5+x^2}+\sqrt{5-x^2}=\frac{4}{\sqrt{5-x^2}}$$

Problem 1.8. Find the value of mn if

$$2^m-2^n=2016$$

Problem 1.9. From the following equation

$$1+\sqrt{3^x}=2^x$$

Find x.

Problem 1.10. If $(x-a)(x-b)=1$ and $a-b+5=0$, then compute the value of

$$(x-a)^3-\frac{1}{(x-a)^3}$$

Problem 1.11. Solve the following equation

$$\frac{1}{x}-4\sqrt{\frac{x+1}{x}}+5=0$$

Problem 1.12. If 5 positive consecutive numbers have a sum of 10^{2018}, then what is the value of the middle number?

Problem 1.13. Find the value of x

$$5^x-5^{x-1}=500$$

Problem 1.14. In a room, there are 144 people. They are joined by n people who are carrying k coins each. When these coins are divided among the $n+144$ people, each person had 2 coins. Find the minimum possible value of $2n+k$.

Problem 1.15. Let x be a real number such that

$$(\sqrt{6})^x-3^x=2^{x-2}$$

Evaluate the following expression

$$\frac{4^{x+1}}{9^{x-1}}$$

Problem 1.16. There are non-zero real numbers a and b so that the roots of x^2+ax+b are $3a$ and $3b$. There are relatively prime positive integers m and n such that $a-b=\frac{m}{n}$. Find $m+n$

Problem 1.17. The sum of the ages of a father and his son is 55. The age of the father is the reverse of the age of the son. What is the age of both the father and the son?

Problem 1.18. Given that

$$\frac{x}{2x^2+5x+2} = \frac{1}{2}$$

Find the value of

$$x + \frac{1}{x}$$

Problem 1.19. Solve for real values of x

$$3^{2x+1} + 4(3^x) - 15 = 0$$

Problem 1.20. Let a and b be positive real numbers satisfying

$$a - 12b = 11 - \frac{100}{a} \quad \text{and} \quad a - \frac{12}{b} = 4 - \frac{100}{a}.$$

Then $a + b = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
